International Journal of Scientific & Engineering Research, Volume 8, Issue 2, February-2017
ISSN 2229-5518

Reducing the Effect of Torque on SISO Gas Turbine System by Applying Suitable H_∞ Controller

Nihal D. Salman[A], Muammel M. Hanon[A]*

[A]Baquba Technical Institute, Middle Technical University, Iraq

Abstract— This paper describes the robust control of single input-single output (SISO) gas turbine system by employing suitable H_∞ Controller. A non-linear model of this system has been taken from literature and developed to a linear model, the state space H_∞ controller is performed using this linearized model. The performance criteria are specified in terms of bandwidth and desired response of the system. The employed H_∞ is compared to a classical Proportional-Integral Drifted (PID) controller, the results show that the H_∞ Controller is best.

Index Terms— *H_∞ Controller, Modeling of SISO gas turbine, Robust control, Simulation by MATLAB.*

1 Introduction

The gas turbines are important and widely employed in transportation and power systems [1]. For developing a control strategy of a gas turbine system there are diverse stages such as; specifying the performance requirements, modeling of the system, designing the controller and validating the controller design by simulating the nonlinear closed loop system [2]. The ability of the closed loop system in responding to a sudden change in the speed is the term of specifying the desired performance for the gas turbine system [3].

The aim of this paper is to apply a suitable H_∞ controller using state space approach to reduce the effect of disturbance (torque) on the system and to achieve the design requirements.

2 Nonlinear Model of the Gas Turbine System

The non-linear modeling is based on the thermodynamic equations in which the behavior of the engine has been described. The major parts of a gas turbine are shown in figure.1 which include; the inlet duct, compressor, combustion chamber, turbine and nozzle or gas deflector. The interactions between these components depend on the physical structure of the engine. The air is drawn into the engine through the inlet duct by the compressor, which compresses it and then delivers it to the combustion chamber. Inside the combustion chamber the air is mixed with fuel and the mixture ignited producing a rise in temperature and hence an expansion occurs for the gases, these gases are exhausted through the engine nozzle or the engine gas-deflector. Moreover, gases pass first through the turbine which is designed to extract sufficient energy from them to keep the compressor rotating, thus the engine is self-sustaining. For gas turbine system there are three quasi polynomial differential equations as presented in equations 1, 2 and 3 respectively [4], [5].

$$\frac{dx_1}{dt} = x_1 \begin{pmatrix} C_{1,1}x_1^{-1}x_2x_3 + C_{1,2}x_1^{-1}x_3 + \\ C_{1,3}x_1^{-1}x_2 + C_{1,4}x_1^{-1} + \\ C_{1,5}x_2x_3 + C_{1,6}x_3 + \\ C_{1,7}x_1^{-.5}x_2^{1.5} + C_{1,8}x_1^{-.5}x_2^{.5} \end{pmatrix} + b_1u = f_1(x_1,x_2,x_3) + b_1u. \tag{1}$$

$$\frac{dx_2}{dt} = x_2 \begin{pmatrix} C_2 + C_{2,1}x_3 + C_{2,2}x_2^{.2857}x_3 + \\ C_{2,3}x_2^{-1}x_3 + C_{2,4}x_2^{-.7143}x_3 + \\ C_{2,5}x_2^{.2857} + C_{2,6}x_2^{-1} + \\ C_{2,7}x_2^{-.7143} + C_{2,8}x_2x_3 + \\ C_{2,9}x_1^{-.5}x_2^{1.5} + C_{2,10}x_1^{-.5}x_2^{.5} \end{pmatrix} + b_2u = f_2(x_1,x_2,x_3) + b_2. \tag{2}$$

$$\frac{dx_3}{dt} = x_3 \begin{pmatrix} C_{3,1}x_2^{2}x_3^{-1} + C_{3,2}x_2^{1.7143}x_3^{-1} + \\ C_{3,3}x_3^{-1}x_2 + C_{3,4}x_2^{.7143}x_3^{-1} + \\ C_{3,5}x_1^{-.5}x_2^{2.5}x_3^{-2} + C_{3,6}x_1^{-.5}x_2^{2.2143}x_3^{-2} \\ +C_{3,7}x_2^{1.5}x_1^{-.5}x_3^{-2} + C_{3,8}x_1^{-.5}x_2^{1.2143}x_3^{-2} \\ +C_{3,9}x_3^{-2} + C_{3,10}x_2^{1.2857}x_3^{-1} + \\ C_{3,11}x_3^{-1} + C_{3,12}x_2^{.2857}x_3^{-1} + \\ C_{3,13}x_2x_3^{-2} + C_{3,14}x_2^{1.2857}x_3^{-2} + \\ C_{3,15}x_2^{.2857}x_3^{-2} \end{pmatrix} = f_2(x_1,x_2,x_3). \tag{3}$$

Where

x_1 is the mass in the combustion chamber m_{Comb}.
x_2 is the turbine total inlet pressure p_3.
x_3 is the rotational speed n.

*Corresponding author: **Muammel M. Hanon**
E-mail address: muammel_mmr_85@yahoo.com

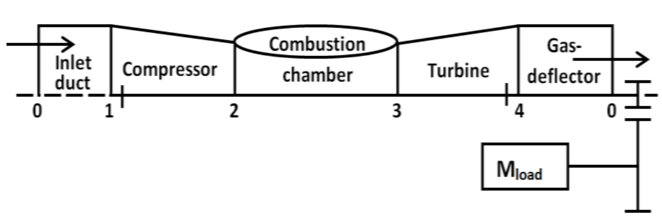

Fig. 1: The main parts of the gas turbine.

3 LINEARIZATION OF NONLINEAR MODEL

The general form of the state space representation of a finite dimensional non-linear time invariant (NLTI) system is [6].

$$\frac{dx(t)}{dt} = f\left(x(t), u(t)\right) \qquad (4)$$

$$y(t) = g\left(x(t), u(t)\right) \qquad (5)$$

To get Liner Time Invariant (LTI) model, it can be represented as:

$$\frac{dx(t)}{dt} = Ax^{\cdot} + Bu \qquad (6)$$

$$y(t) = Cx + Du \qquad (7)$$

The output of the control system (y) is rotational speed n. the disturbance is M_{load} (torque).
Let the pressure p_3 is held constant

$$x_2 = x_2^0 = p_3^0 = 223587.2 \text{ Pa. [3].}$$

Let the right side of equations (1, 2, 3) denoted by f which differentiated to x1 and x2 hence, the nominal value of x is substituted in it then the constant matrices A, B, C and D that have the dimensions $n \times n$, $n \times m$, $r \times n$ and $r \times m$ respectively can be found as [4].

$$A = \partial f / dx|_{x^0, u^0} , \qquad B = \partial f / du|_{x^0, u^0}$$

$$C = \partial f / dx|_{x^0, u^0} , \qquad D = \partial f / du|_{x^0, u^0}$$

The matrices B does not depend on the state vector, which is a real constant. Then

$$B = \begin{bmatrix} 1 \\ 0 \\ \end{bmatrix}$$

The matrices C = [0 1] and D = [0]
The nominal values of x are [2, 3].

$$x^0 = \left[0.00528 \text{ kg} \;\; 750\frac{1}{s} \right]^T$$

$$\text{and} \quad u^0 = 0.009913$$

By using MATLAB 7.8 program, the state space matrices A,B,C and D were found to be:

$$A = \begin{bmatrix} 0.0005 & 0.0253 \\ 0.0017 & -7.5329 \\ & \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ \end{bmatrix}, C = \begin{bmatrix} \\ 0 & 1 \\ \end{bmatrix}, D = 0.$$

The nominal function of gas turbine system become as:

$$G_{nom}(s) = \frac{.0017244}{(s + 7.533)(s - .0004659)} \qquad (8)$$

The dimensionless can be used to decrees the huge difference in order between x_1 and x_2.

4 PERFORMANCE REQUIREMENTS

The design requirements that were compatible with this paper could be summarized as follow [7].

- Good and swift disturbance rejection.
- The closed loop system is stable at all operation conditions.
- The settling time should be less than 8 seconds.
- The bandwidth of closed loop system is about (0.2-20 rad/sec).

5 H_∞ CONTROLLER DESIGN AND EVALUATION

The first step in the design procedure should start by acquiring augmented plant ***P(s)*** for the given system nominal transfer function $\boldsymbol{G_{nom}(s)}$ with the selected weighting function. The second step is to obtain iterative procedure to find the value of upper bound of the $\boldsymbol{H_\infty}$ norm. This would ensure the existence of sufficient conditions obtaining a stabilizing controller. The evaluation procedure included two parts. The first part is to determine the responses in the frequency domain which includes singular value plots for both open and closed loop systems to check the design requirements, the plots for the sensitivity, complementary sensitivity and control sensitivity functions. The second part is to determine the responses in the time domain which includes the plant output response. The transfer function of gas turbine system has a pole in the right side of the complex plane, this will indicate that the system is unstable. The state feedback by using pole

IJSER

placement has utilized and then H_∞ controller is applied to the system. The desired selected closed loop pole is based on the required bandwidth of plant, trial and error was used to select the pole, this pole will equal [-0.08, -9]. Using MATLAB program the state feedback controller K(s) was obtained as $K=10^3 *(0.0015, -6.3409)$ and the new transfer function of gas turbine system given as

$$G_s(s) = \frac{0.0017244}{(s+9)(s+0.08)} \quad (9)$$

6 H_∞ CONTROL PROBLEM FORMULATION

The design goals are reflected by weighting the appropriate closed-loop transfer functions in the H_∞ synthesis. The standard H_∞ control problem is shown in figure 2, and represented as

$$P(s) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (10)$$

The plant ***P(s)*** called augmented plant which includes the plant model and weighting function. The objective of H_∞ control is to calculate $K(s)_\infty$ thus $F_l(P,K)$ is minimized according to the H_∞ ***norm*** of the transfer function matrix $F_l(P,K)$ that identified by the following equation [8], [9].

$$T_{zw\infty} = F_l(P,K)_\infty = max_\omega \bar{\sigma}\left(F_l(P,K)(j\omega)\right) \le \gamma \quad (11)$$

Where $\gamma \in R$, $F_l(P,K) =$

$$P_{11}(s) + P_{12}(s)k(s)\left[I - P_{22}(s)k(s)\right]^{-1} P_{21}(s)$$

The closed loop from the external w to the regular is output z and $\sigma\ (F_l(P,K)(j\omega))$ is the largest singular value of $F_l(P,K)$ at the frequency ω.

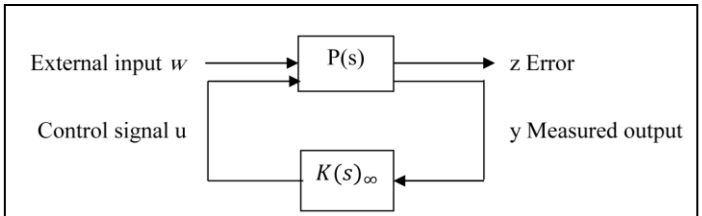

Fig. 2: standard H_∞ control problem.

It is necessary to formulate the H_∞ problem in terms of the sensitivity function $S = (1 + GK)^{-1}$, complementary sensitivity function $T = GK(1 + GK)^{-1}$ and control sensitivity function $R = K(1 + GK)^{-1}$ [10].

However, there is a tradeoff between these two performance criteria which can make the feedback design difficult, to assist in making this tradeoff, it is usual to weight the relevant sensitivity function S(s) therefore the design is able to emphasize the appropriate properties at the different frequencies. The most common approach is to keep **|S (jω)|** small at low frequencies, and keep **|T (jω)|** small at high frequencies. Now for the gas turbine system $F_l(P,K)$ is chosen to be a function of S, T and R. The weights are the major design parameters in any H_∞ design. The following guidelines summarize how to choose the weights [11].

1. Choose the transfer function (filter) $W_p(s)$ as a low frequency weight to insure good tracking and output disturbance rejection.
2. Choose $W_u(s)$ as a weight on the actuator inputs to prevent actuator saturation and achieve robustness to plant additive perturbations.
3. Choose $W_o(s)$ as a high frequency weight to limit the closed loop bandwidth, ensure robustness to plant output multiplicative perturbations.
4. Keep the degree of the weights as low as possible because the degree of the controller generally equals the degree of the plant plus the degree of the weights.

To construct the augmented system as shown in figure 3, let z be the regulated output vector, w be the external input vector, x the state vector of the augmented system.

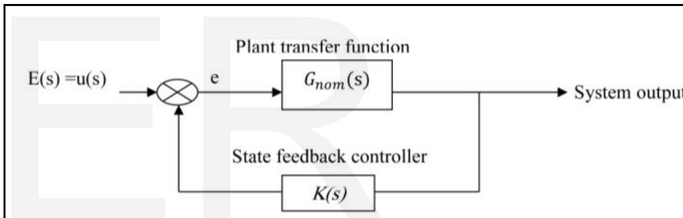

Fig. 3: block diagram of state feedback controller.

There are four conditions for stability and performance can be derived as follows:

i. Nominal stability (NS): for figure 4 assuming (Δ=0), all poles of the closed loop T_{zw} must be lie in left half complex plane to satisfy the nominal stability.

ii. Robust stability (RS): condition for robust stability is

$$RS \Leftrightarrow |T| < |1/W_o|, \forall\omega$$

iii. Nominal performance (NP): To obtain the nominal performance assume Δ=0 then condition for nominal performance becomes

$$NP \Leftrightarrow |S| < |1/W_P|, \forall\omega$$

iv. Robust performance (RP): the condition for robust performance is

$$RP \Leftrightarrow max_\omega\left(|W_P S| + |W_o T|\right) < 1$$

ISSN 2229-5518

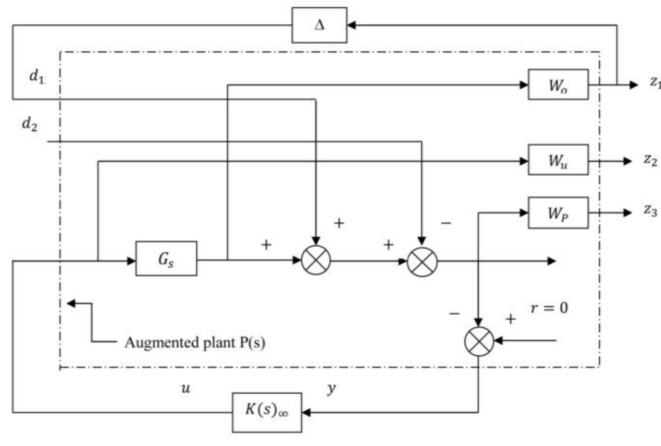

Fig. 4: Augmented system for H_∞ problem formulation.

7 Simulation Results and Discussion

The weighting functions are chosen by trial and error. The following set weighting function is chosen to reflect the desired performance

$W_p = \frac{0.4s+0.22}{s+0.0003}$ (Good tracking and output disturbance rejection)

$W_u = 10^{-4}$ (Prevent saturation & introduce robustness to additive uncertainties)

The Uncertainty weighting filter was chosen to be in the following form:

$W_o = \frac{3s+20.81}{s+24.19}$ (To limit the closed loop bandwidth & introduce robustness to plant output multiplicative uncertainties)

The state space representations are found to be:

$$A_{aug} = \begin{bmatrix} -0.6 & 2.0135*10^3 & 0 & 0 & 0 \\ 0.001 & -7.5329 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & -24.1935 & 0 \\ 0.5 & 0 & 0 & 0 & -3*10^{-4} \end{bmatrix}$$

$$B_{1_{aug}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.5 & -0.5 \end{bmatrix}, B_{2_{aug}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{1_{aug}} = \begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0.2599 \end{bmatrix},$$

$$C_{2_{aug}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_{11_{aug}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & -0.5 \end{bmatrix}, D_{12_{aug}} = \begin{bmatrix} 0 \\ 5.3879*10^{-4} \\ 0 \end{bmatrix},$$

$$D_{21_{aug}} = \begin{bmatrix} 0 & 0 \end{bmatrix}, D_{22_{aug}} = 0$$

- A Value of γ_{min} of the optimal solution equals to 1.2482 and the H_∞ norm of the closed loop transfer function $\|T_{zw}(S)\|_\infty$ equals to 1.2482 this means that the controller is admissible.
- The H_∞ robust controller is suboptimal with transfer function given as:

$$k(s)_\infty = \frac{121868.7452(s+0.08)(s+9)(s+24.19)}{(s+24.1)(s+0.0003)(s^2+26.45s+187)}$$

- The pole of closed loop are found to be:
  -0.0003, -24.1935, -0.0800, -1.3863, -9, -10.3345, -14.7327,-24.0937
  All the poles of closed loop system lie in the left side of the complex plane, so the nominal stability is guaranteed.
- The singular value of open loop plant, stabilized plant system and state feedback H_∞ controlled systems are all shown in figure 5. The maximum singular value of three loop systems that obtained are (0.4804 db,

0.0024 db, 1.24 db) respectively. As it can be seen, the closed loop of state feedback H_∞ controlled system is stable over required range until the rolling will be start at (10 rad/sec).

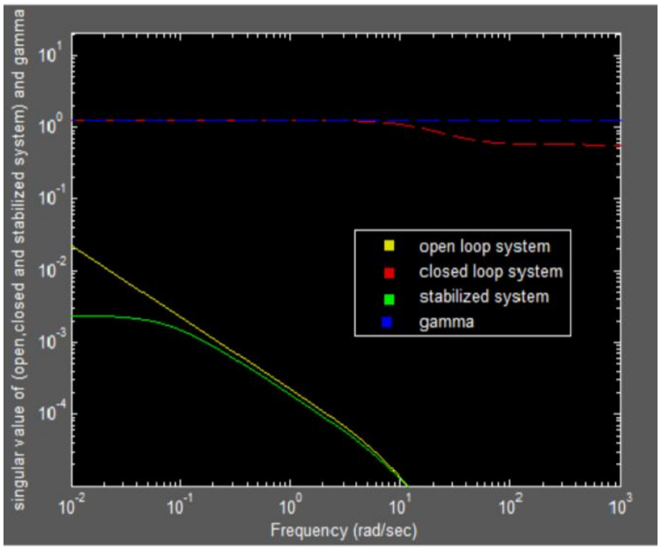

Fig. 5: Singular value for open plant, stabilized system, controlled and gamma value.

- The conditions for nominal performance NP, robust stability RS and robust performance RP can be summarized as follows:
- Nominal performance NP: In figure 6, the singular value of sensitivity function S which gained is (1.03 db at 6.12 rad/sec) and $| W_P |$ equals (7.92 db at 2.15 rad/sec). In this figure, a good coverage for sensitivity function by its bound can be shown. At the same time, the sensitivity function keeps a smooth shape without resonant peak.

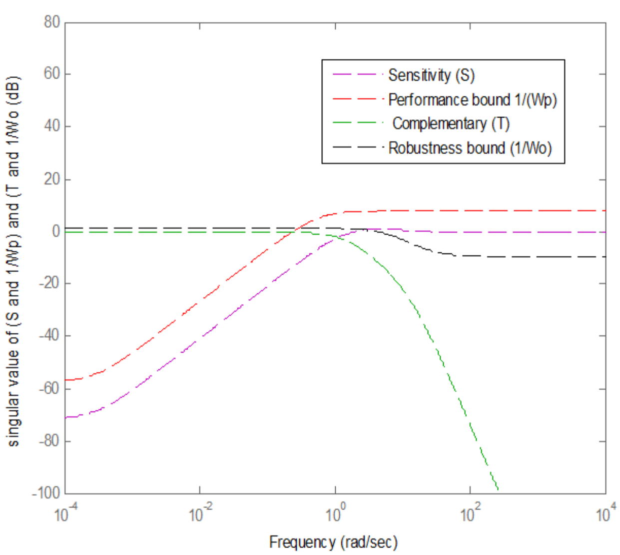

Fig. 6: Singular value of S, T and their performance bound.

- In figure 7, the singular value for the control sensitivity function R that found is (76.8 db at 9.12 rad/sec) and performance bound $| W_u |$ equals (80 db at 4.24 rad/sec), it can be seen in this figure that control sensitivity function is totally surrounded by its performance bound.

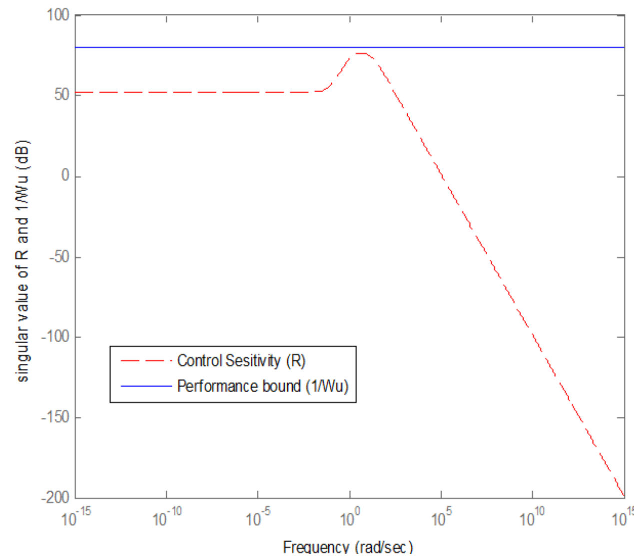

Fig. 7: Singular value of R and its bound.

- Robust stability RS: In figure 6, the singular value of complementary sensitivity function T that obtained is (-0.336 db at .429 rad/sec) and $| W_o |$ equals (1.92 db at .429). A very sharp roll off to the complementary sensitivity function is achieved by the H_∞ controller, this is an indication that the high frequency of any uncertainty or noise is being attenuated. It can be seen that the system has been yield to the condition $max|W_P S| + |W_o T| < 1$ and the nominal performance becomes satisfied as shown in figure 8.

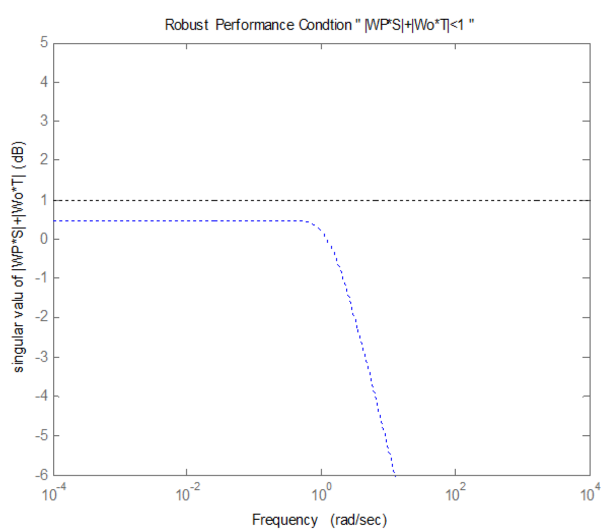

Fig. 8: Robust performance test.

- The time domains results were illustrated in figure 9 and 10. In figure 9, the system responses to step input which was applied at the reference r. whereas in figure 10 the system responses to 0.1 step inputs, this figure offered a good rejection for disturbance with settling time of (5 sec) and the response has no overshoot.

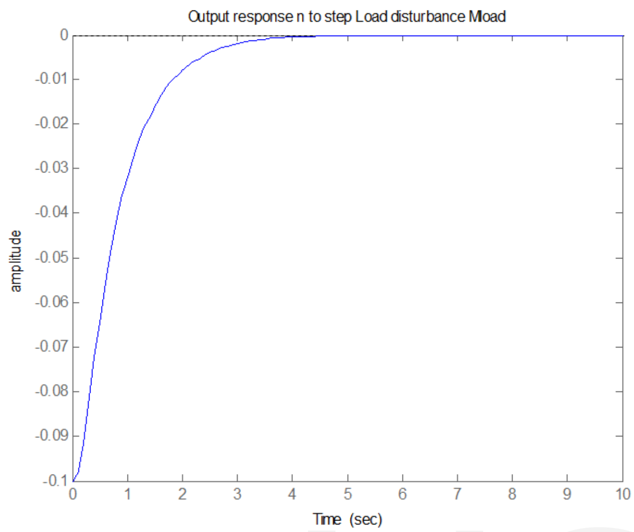

Fig. 9: output response (n) in per unit pu for step reference input.

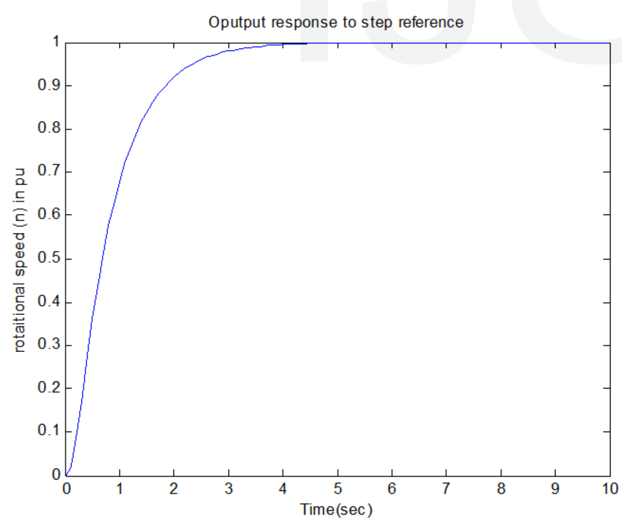

Fig. 10: Output response (n) in pu for 0.1 step disturbance (Mload).

- Figure 11 illustrates the output response for PID controller. The comparison between the figures 10 and 11 shows that the output response for H_∞ controller is best and has salting time less than PID controller.

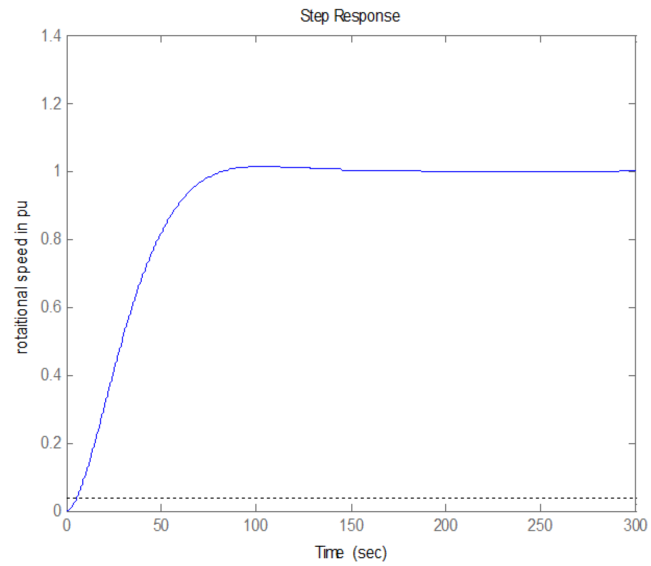

Fig. 11: output response (n) in pu for step reference input for PID controller.

8 Conclusion

In this paper the new H_∞ controller is presented based on state space approach using MATLAB environment. The state feedback (pole placement) approach has been employed to stabilize the system by applying H_∞ controller. It has been shown that H_∞ controller gives faster response than the PID controller in addition to obtain a robust control system. Less settling time has been investigated by picking appropriate weighting function, it has been done by the method of try and error. For choosing suitable weighting filters, design experience and knowledge of the plants are the most assistant factors.

References

[1] C. Soares, "Gas Turbine: A Handbook of Air, Land and Sea applications", *BH Publications*, 2007.

[2] H. W. Gomma and D. H. Owens "Robust Control of Gas Generator in a 1.5 MW Gas Turbine Engine", *Proceedings of the IEEE International on Control Application*, Hawaii, August, 1999.

[3] D. Biss, "Gas Turbine control using polynomial H_∞ design techniques", *Ph.D. thesis*, University of Strathclyde, UK, 1991.

[4] P. Ailer, "Nonlinear Mathematical Modeling and Control Design Developed for Gas Turbine", *Proceedings of the 7th Mini Conference on Vehicle System Dynamics, Identification and Anomalies*, pages 465–472, Budapest University, Hungary, 2000.

[5] P. Ailer, "Modeling and Nonlinear Control of a Low Power Gas Turbine", *Ph. D. Thesis*, Budapest University of Technology and Economics, Hungary, 2002.

[6] H. L. Trentelman, A. A. Stoorvogel and M. Hautus "Control Theory for linear system", *Book*, Netherland, 2002.

[7] O.F. Qi, P.J. Gawthrop and N.R.L. Maccallum,"Meeting the Performance Requirement of a Single Spool Gas Turbine Engine Using a Gain Scheduled Controller", *Proceedings of the IEEE International on Intelligent Control*, Glasgow, UK, pp. 385-390, 1992.

[8] k. Zhou and J.C. Doyle, "Essential of Robust Control", *Prentice Hall Inc.*, 1999.
[9] J. Burl, "Linear optimal control", *Addison Wesley Longman Inc.*, Menlo Park, California, USA, 1999.
[10] The mathworks,"Robust control toolbox", *Users guide for use with MATLAB*, V.2.08, 2001.
[11] S. Skogestad and I. Postlethwaite, "Multivariable Feedback Control Analysis and Design", *John Wiley and Sons Ltd*, England, 2005.

IJSER